Orientifold Planar Equivalence: The Quenched Meson Spectrum

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A numerical study of Orientifold Planar Equivalence is performed in SU(N) Yang-Mills theories for $N = 2, 3, 4, 6$. Quenched meson masses are extracted in the antisymmetric, symmetric and adjoint representations for the pseudoscalar and vector channels. An extrapolation of the vector mass as a function of the pseudoscalar mass to the large-N limit shows that the numerical results agree within errors for the three theories, as predicted by Orientifold Planar Equivalence. As a byproduct of the extrapolation, the size of the corrections up to $O(1/N^3)$ are evaluated. A crucial prerequisite for the extrapolation is the determination of an analytical relationship between the corrections in the symmetric and in the antisymmetric representations, order by order in a $1/N$ expansion.

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1. Introduction

Orientifold Planar Equivalence [1] is a powerful analytical tool that establishes the equality of certain observables (among which, meson masses) in two classes of gauge theories at $N = \infty$, N being the number of colours. This equivalence is of particular interest when the two gauge theory families it relates are SU(N) with N_f (anti)symmetric Dirac fermion flavours and SU(N) with N_f Majorana flavours in the adjoint representation. In fact, this special case enables us to transcribe SUSY results to QCD [2], provided that the latter is close (in the sense of the $1/N$ expansion) to its large-N limit. Some issues related to the equivalence, like e.g. the size of the finite N corrections, mandate an *ab-initio* calculation. A full dynamical calculation is expensive from a computational point of view. In fact, the cost of the computation is brought almost entirely by the inversion of the fermionic matrix, which for theories with two-index representation fermions requires $O(N^4)$ operations. Before undertaking a full dynamical study, it is convenient to consider the quenched case as a prototype example. Although for theories in two-index representations the quenched theory and the dynamical theory do not coincide in the large-N limit, Orientifold Planar Equivalence still holds in the quenched case, which then becomes a useful toy model to study techniques that could be used in the dynamical investigation. In this spirit, a first paper appeared in which the chiral condensate was shown to coincide in the large-N limit of the adjoint, symmetric and antisymmetric representations [3]. A crucial point in that work was the separation of the corrections in even and odd powers of $1/N$ by combining the numerical results in the symmetric and antisymmetric representations. As a result, a precise extrapolation to the large-N limit could be performed and Orientifold Planar Equivalence was verified to a high degree of accuracy. Moreover, this technique allowed a computation of corrections up to $O(1/N^3)$ in the (anti)symmetric representation while still using a reasonably small value of N. In this work, we will perform a similar study for the quenched meson spectrum. First, we will show that there is a simple order by order relationship between the coefficients in a large-N expansion of amplitudes in correlation functions and of spectral masses in the symmetric and antisymmetric representations. Then, we shall use a simple chiral ansatz to establish a relationship between the mass of the vector meson m_V and the mass of the pseudoscalar meson m_{PS} at finite N, for N ranging from two to six, in the adjoint, in the symmetric and antisymmetric representations. Finally, using the analytical relationships between the corrections, the chiral ansatz will be extrapolated to the large-N limit and Orientifold Planar Equivalence proven to hold. The results we will present in this work have already appeared in [4], to which we refer for a more detailed discussion.

2. Meson masses for two-index representation fermions in the $1/N$ expansion

We extract meson masses from the large distance exponential decay of the meson correlation functions. For the fermion sources we use the Wilson formulation, in terms of which the Dirac operator is given by

$$D_{xy} = (m+4r)\delta_{xy} - K_{xy} \,, \tag{2.1}$$

with m the bare quark mass and

$$K_{xy} = -\frac{1}{2} \left[(r - \gamma_\mu) R\left[U_\mu(x)\right] \delta_{y,x+\hat{\mu}} + (r + \gamma_\mu) R\left[U_\mu^\dagger(y)\right] \delta_{y,x-\hat{\mu}} \right] \,. \tag{2.2}$$

In the previous formulae, x and y are site indices and the γ_μ the Dirac Γ matrices in Euclidean space. $R\left[U_\mu(x)\right]$ is the link variable stemming from x in the direction $\hat{\mu}$ expressed in the representation R. In our case, R can be the adjoint (Adj), the two-index symmetric (S) or the two-index antisymmetric (AS) representation. For the representation R, meson correlators take the form

$$C^R_{\Gamma_1\Gamma_2}(x,y) = r_R \left\langle \mathrm{tr}_R\left(D^{-1}_{yx}\Gamma^\dagger_1 D^{-1}_{xy}\Gamma_2\right)\right\rangle_{YM} , \tag{2.3}$$

where tr_R indicates the trace over the color indices in the representation R and Γ_1 and Γ_2 are combinations of Γ matrices associated with the quantum numbers J^{PC} of the mesons. With the choice $r_R = 1$ for $R = \mathrm{S/AS}$ and $r_R = 1/2$ for $R = \mathrm{Adj}$, in the large-N limit we have the equality of correlators in these three representations. The factor $r_R = 1/2$ in the adjoint representation can be understood considering that in our formulation we are always taking Dirac flavours in the adjoint, and one Dirac flavour is equivalent to two Majorana flavours.

The formal proof of the equality of the correlators can be sketched as follows. We start expanding the correlators in Wilson loops:

$$\frac{1}{N^2}C^R_{\Gamma_1\Gamma_2}(x,y) = \frac{r_R}{N^2}\sum_{\mathscr{C}\supset(x,y)}\alpha_\mathscr{C}\left\langle\mathrm{tr}_R W_\mathscr{C}\right\rangle , \tag{2.4}$$

$\mathscr{C}$ being a closed curve that contains both x and y. The coefficient $\alpha_\mathscr{C}$ in (2.4) does not depend on the representation. Writing explicitly the traces in two-index representations in terms of the trace of the fundamental representation yields

$$\frac{1}{N^2}C^{\mathrm{S/AS}}_{\Gamma_1\Gamma_2}(x,y) = \frac{1}{2}\sum_{\mathscr{C}\supset(x,y)}\alpha_\mathscr{C}\frac{\langle[\mathrm{tr}\,W_\mathscr{C}]^2\rangle\pm\langle\mathrm{tr}[W^2_\mathscr{C}]\rangle}{N^2} , \tag{2.5}$$

$$\frac{1}{N^2}C^{\mathrm{Adj}}_{\Gamma_1\Gamma_2}(x,y) = \frac{1}{2}\sum_{\mathscr{C}\supset(x,y)}\alpha_\mathscr{C}\frac{\langle|\mathrm{tr}\,W_\mathscr{C}|^2\rangle-1}{N^2} . \tag{2.6}$$

Note that the factors of $1/2$ in the two previous formulae have a different origin: for the S/AS, it comes when expressing the trace in the two-index representations in terms of the group element in the fundamental representation, while for the adjoint is the factor r_R. At large N, we can use factorisation and neglect the subleading terms, obtaining

$$\frac{1}{N^2}C^{\mathrm{S/AS}}_{\Gamma_1\Gamma_2}(x,y) = \frac{1}{2}\sum_{\mathscr{C}\supset(x,y)}\alpha_\mathscr{C}\frac{\langle\mathrm{tr}\,W_\mathscr{C}\rangle\langle\mathrm{tr}\,W_\mathscr{C}\rangle}{N^2} , \tag{2.7}$$

$$\frac{1}{N^2}C^{\mathrm{Adj}}_{\Gamma_1\Gamma_2}(x,y) = \frac{1}{2}\sum_{\mathscr{C}\supset(x,y)}\alpha_\mathscr{C}\frac{\langle\mathrm{tr}\,W_\mathscr{C}\rangle\langle\mathrm{tr}\,W^\dagger_\mathscr{C}\rangle}{N^2} . \tag{2.8}$$

Finally, the equality

$$\lim_{N\to\infty}\frac{1}{N^2}C^{\mathrm{S/AS}}_{\Gamma_1\Gamma_2}(x,y) = \lim_{N\to\infty}\frac{1}{N^2}C^{\mathrm{Adj}}_{\Gamma_1\Gamma_2}(x,y) \tag{2.9}$$

follows from charge conjugation invariance, which dictates $\langle\mathrm{tr}\,W^\dagger_\mathscr{C}\rangle = \langle\mathrm{tr}\,W_\mathscr{C}\rangle$. This last step in Yang-Mills theories does not present the conceptual issues discussed in [5, 6] for the dynamical

case. The equality of the meson spectra in the three theories follows from the equality of the correlators.

Correlators of mesons in the adjoint representation are expected to be expandable in a power series of $1/N^2$. As for the other two representations, since in the symmetric and antisymmetric representations the static sources introduce $1/N$ effects, there is no reason not to expect $1/N$ corrections in the meson mass spectrum. Assuming that there is no accidental degeneracy at large-N in the theory with fermions in the antisymmetric and in the symmetric[1], the masses m_R and the amplitudes A_R in correlators of fermion bilinears in these two representations can be shown to be related by

$$A_S^j(N) = A_{AS}^j(-N) \qquad \text{and} \qquad m_S^j(N) = m_{AS}^j(-N) \qquad (2.10)$$

for states j related by Orientifold Planar Equivalence. Order by order in a $1/N$ expansion, this implies that even-power terms in $1/N$ in the symmetric and antisymmetric representation have the same coefficient, while odd-power terms have opposite coefficients. This means that the combinations

$$M^j = \left(m_S^j + m_{AS}^j \right)/2 \qquad \text{and} \qquad \mu^j = N \left(m_S^j - m_{AS}^j \right)/2 \qquad (2.11)$$

have an expansion in powers of $1/N^2$.

3. Numerical results

We generated gauge configurations for values of N ranging from two to six using the standard Wilson action. Both the generation of the configurations and the analysis of the correlators in two-index representations were performed using the purposely developed HiRep code [7]. The technical details of the simulations have been given in [4]. Here we remark that we simulate at the same β values as in Ref. [3], where the couplings were chosen so that the gauge theories at various N are at a common value of the lattice spacing $a = 1/(5T_c)$, T_c being the deconfinement temperature of the pure gauge system [8].

After extracting m_V and m_{PS}, we have determined the chiral limit value of m_V (denoted as m_V^χ) using the chiral ansatz

$$m_V(m_{PS}) = m_V^\chi + B m_{PS}^2 . \qquad (3.1)$$

Our results and the corresponding fits are displayed in Figs. 1, 2 and 3 respectively for the adjoint, symmetric and antisymmetric representations. We note that there is a well-defined hierarchy for m_V at fixed m_{PS}, with the symmetric mass being the highest and the antisymmetric being the lowest. At fixed representation, the vector mass in the antisymmetric has the largest variation with N, while there is little or no variation with N for the adjoint vector masses.

The large-N extrapolation has been performed for the quantity $m_{V,Adj}^\chi$ and for the quantities M_V^χ and μ_V^χ (see Eqs. (2.11)) using in all cases only the leading term and the $1/N^2$ correction. The

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[1]In Quantum Field Theory, degeneracies are due to a symmetry, and there is no symmetry that could generate degeneracies in these theories.
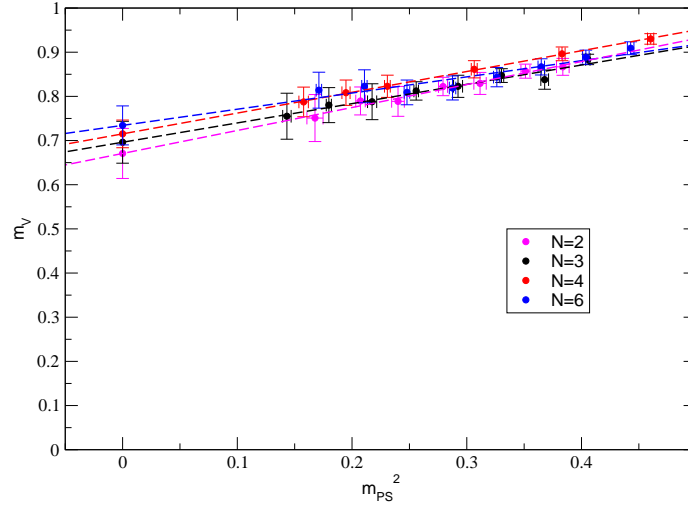
Figure 1: m_V as a function of m_{PS}^2 for the adjoint representation at the values of N shown. The solid lines are chiral fits to the data.
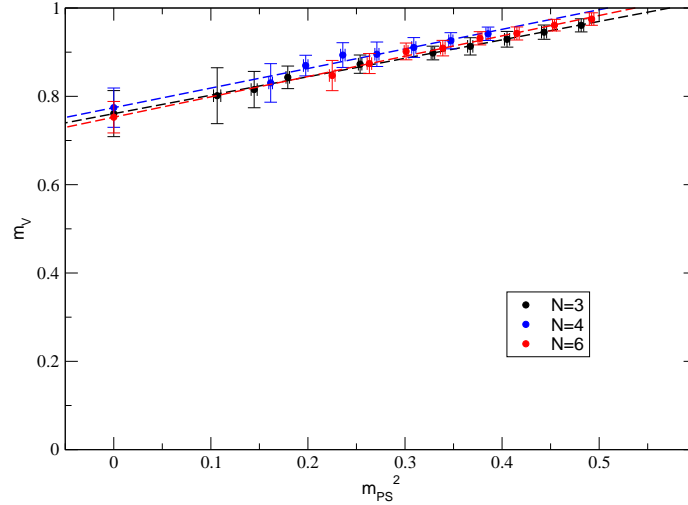
Figure 2: m_V as a function of m_{PS}^2 for the symmetric representation at the values of N shown. The solid lines are chiral fits to the data.
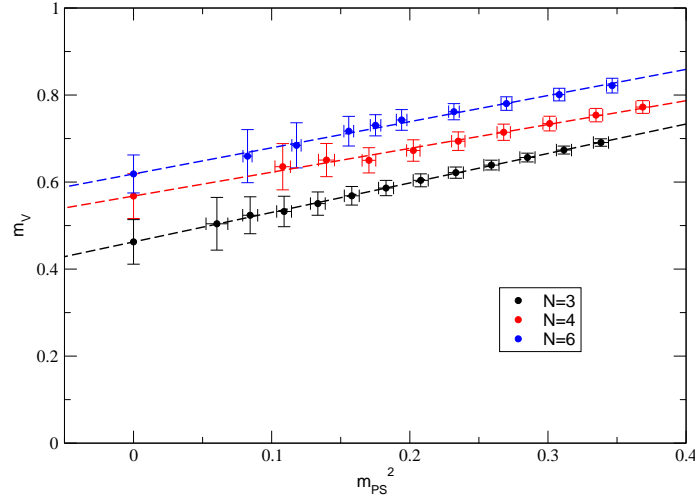
quality of the fits is displayed in Fig. 4. The values of $m_{V,S}^{\chi}$ and $m_{V,AS}^{\chi}$ as a function of $1/N$ are obtained inverting Eqs. (2.11). Our results are:

$$m_{V,\text{Adj}}^{\chi} = 0.736(17) - \frac{0.28(11)}{N^2} \ , \tag{3.2}$$
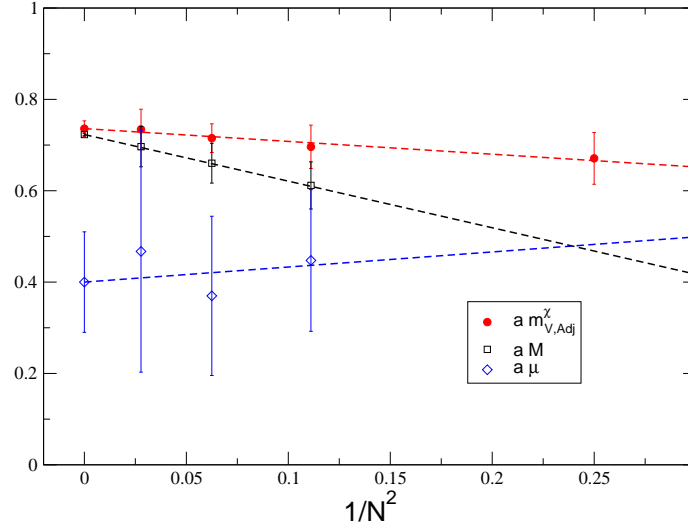
$$m_{V,S}^{\chi} = 0.723(27) + \frac{0.40(11)}{N} - \frac{1.02(32)}{N^2} + \frac{0.3(1.2)}{N^3} \ , \tag{3.3}$$

$$m_{V,AS}^{\chi} = 0.723(27) - \frac{0.40(11)}{N} - \frac{1.02(32)}{N^2} - \frac{0.3(1.2)}{N^3} \ . \tag{3.4}$$

The equality of the large-N results for the three representations is a prediction of Orientifold Planar

Figure 3: m_V as a function of m_{PS}^2 for the antisymmetric representation at the values of N shown. The solid lines are chiral fits to the data.

Figure 4: Large-N extrapolation of the adjoint mass and of the combinations M and μ, all in the chiral limit.

Equivalence. We remark that through the separations of corrections in even and odd powers of $1/N$ in the symmetric and antisymmetric representation, we were able to get an handle on the size of the corrections up to $O(1/N^3)$. Numerically, Orientifold Planar Equivalence can be verified for any value of m_{PS} in the chiral region [4].

4. Conclusions

Our work should be regarded as an illustration of a possible strategy for a numerical proof of Orientifold Planar Equivalence. We have shown that it is possible to combine the data in the symmetric and antisymmetric representation for extracting precise values in the large-N limit from

data at moderately small values of N. The next step in this investigation is the simulation of the dynamical theories.

Acknowledgments

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